Perfect substitutes and income compensation

Consider a utility function $u = 3x_1 + x_2$. The prices in this economy are $p_1 = \$7$ and $p_2 = \$4$. The price of good 1 increases to \$8. How does the optimal choice change in this case? If one wanted to maintain the purchasing power from the first case, how would the income would change? And what if one wanted to maintain the level of utility? Justify.

Solution

The optimal demands of a function with perfect substitutes depend on the ratio of prices compared to the substitution coefficient of one good for another.

$$\frac{p_1}{p_2} = \frac{7}{4}$$

$$\frac{u'x_1}{u'x_2} = \frac{3}{1}$$

With this, we see that:

$$\frac{7}{4} < \frac{3}{1}$$

Therefore, the optimum is $(M/p_1, 0) = (M/p_1, 0)$. If the price of good 1 increases, the optimal basket remains the same, as it still holds that

$$\frac{8}{4} < \frac{3}{1}$$

The previous utility is:

$$u(x_1^m, x_2^m) = \frac{3M}{p_1} + 0 = \frac{3M}{7} + 0$$

The new utility is:

$$u(x_1^m, x_2^m) = \frac{3M}{p_1'} + 0 = \frac{3M}{8} + 0$$

To maintain the purchasing power, it is necessary to increase M so that with the new income, one can buy $M/7$, which was possible previously:

$$\frac{M'}{8} = \frac{M}{7}$$

$$M' = \frac{8M}{7}$$

And to maintain the utility:

$$\frac{3M'}{8} = \frac{3M}{7}$$

$$M' = \frac{8M}{7}$$

In both cases, an increase in income is necessary such that the new income is $\frac{8}{7}$ of the previous, that is, 14% more. This way, one could reach the previous basket and the previous level of utility.